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COOPERATIVE EFFECTS IN LIGHT SCATTERING FROM AEROSOL CLUSTERS AND SURFACE ASSEMBLIES

1979 Final Report

Richard A. Farrell
Robert H. Ardreo
Jerry A. Krill

June 1980

JOHNS HOPKINS UNIVERSITY
Applied Physics Laboratory
Laurel, Maryland 20810

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Vector stochastic variational principles are derived for the statistics of the scattering of a plane electromagnetic wave from inhomogeneous and anisotropic conducting, dielectric objects or surfaces with arbitrary, random electrical and geometrical characteristics. These stochastic variational formulations are based on deterministic variational principles of the general form $T = (4\pi)^{-1}N_1N_2/D$, where T is a component of the far-field scattering amplitude and N_1, N_2, and D are integrals involving the fields or currents at

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20. the scatterers. The nonstochastic nature of the incident field allows the statistical moments of T and of the differential scattering cross section $|T|^2$ to be expressed as the vector stochastic variational principles $\langle T^n \rangle = (4\pi)^{-n} \langle N_1{}^n \rangle \langle N_2{}^n \rangle / \langle D \rangle$ and $\langle |T|^{2n} \rangle = (4\pi)^{-2n} \langle |N_1|^{2n} \rangle \times \langle |N_2|^{2n} \rangle / \langle |D|^{2n} \rangle$ for arbitrary scatterer statistics. They are readily observed to be inherently simpler than direct averages such as $\langle T^n \rangle = (4\pi)^{-n} \langle N_1{}^n N_2{}^n / D^n \rangle$, and should allow practical application of variational techniques to random scattering problems.

PREFACE

This work was supported in part by the Department of the Navy (Naval Sea Systems Command) and by the Department of the Army (Armament Research and Development Command) under Contract N00024-78-C-5384. The U. S. Army Armament Research and Development Command, through its Chemical Systems Laboratory, provided short term support to allow our wave-scattering program to start in FY79. The Army Research Office will support our efforts in this area from 1 January to 31 December 1980 with funding optional for the second and third years. This report covers progress made from 15 August through 31 December 1979. A more detailed description of our progress is contained in the manuscript which is appended and has been accepted for publication in IEEE Transactions on Antennas and Propagation.

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Two of the authors (J. A. Krill and R. H. Andreo) wish to thank Drs. R. A. Farrell and R. W. Hart for suggesting this problem and providing valuable insights throughout the course of this work, and Dr. M. R. Feinstein for numerous spirited discussions of variational methods. We would also like to thank Dr. L. S. Taylor of the University of Maryland, Electrical Engineering Department for helpful advice concerning the statistics of scattered fields.

CONTENTS

COOPERATIVE EFFECTS IN LIGHT SCATTERING FROM AEROSOL *CLUSTERS AND SURFACE ASSEMBLIES*

1. INTRODUCTION

1.1 Objectives

The long-range objective of this program is to acquire a better understanding of cooperative effects, i.e., interference and multiple scattering, and polarization in the scattering and absorption of electromagnetic radiation by random surfaces and random collections of particles. This objective is to be achieved by developing stochastic variational principles for calculating wave scattering and using specialized-trial cases to test the efficacy of these principles.

1.2 Progress

During the short span of the reporting period, we have completed the development of vector stochastic variational principles for electromagnetic plane wave scattering from perfectly conducting objects and rough surfaces and from conducting dielectrics which are generally inhomogeneous and anisotropic. The parameters describing the scatterers may be deterministic or random with completely arbitrary statistics. Specifically, the stochastic vector principle for conducting dielectrics was extended to include anisotropic materials with no symmetry restrictions on the conductivity and permittivity tensors. In addition, a stochastic variational principle was derived for polarized plane wave scattering from a randomly rough planar surface which is perfectly conducting. This formulation incorporates a half-space dyadic Green function. Finally, a variational principle for closed, perfectly conducting scatterers was recast into stochastic form. These variational principles are for the statistical moments and probability density functions of T and $|T|^2$, where $T \equiv \hat{e}_s \cdot \vec{T}$ is the component of the normalized far-field vector scattering amplitude $\vec{T}$ along with an arbitrary polarization direction $\hat{e}_s$. A detailed description of this work is given in Sections 2 through 6 which follow. These sections are an exact duplication of a report by Jerry A. Krill and Robert H. Andreo of the Applied Physics Laboratory, The Johns Hopkins University.

1.3 Future Work

Our work in the near term will involve the development of a completely general stochastic vector variational principle for scatterers with inhomogeneous and anisotropic magnetic, as well as dielectric and conductive, properties. Subsequent efforts will then be directed towards variational

calculations for specific scattering problems. This work will be supported by the Army Research Office.

2. VECTOR STOCHASTIC VARIATIONAL PRINCIPLES FOR ELECTROMAGNETIC WAVE SCATTERING

This is a report by J. A. Krill and R. H. Andreo of The Johns Hopkins University, Applied Physics Laboratory, Johns Hopkins Road, Laurel, Maryland 20810.

2.1 History

During the past three decades numerous approximation techniques have been applied to model electromagnetic wave scattering from surfaces and media.[1-3] Variational methods[4-6] have been of value in studies of deterministic scattering and propagation and are recognized as having several important advantages.[4-10] The primary one is the cancellation of first order errors arising from the initial estimate of the (generally unknown) field on or within the scatterer. This cancellation will lead to accurate results if the estimate is already a good approximation. Thus, it is fortunate that the variational principle for scattering contains a built-in indicator of accuracy and region of validity.[7] Also, in certain cases an error analysis may be performed which provides a useful error bound for the chosen trial estimate.[11] Until recently,[7] however, difficulties inherent in calculating the statistics of the scattered field using the variational principle have precluded more than limited application to stochastic problems.[7,8,11] The difficulties occur in evaluating the statistical moments of the ratio of complex integrals, a characteristic of variational scattering calculations.

In 1977, Hart and Farrell[7] recast a scalar variational principle into a form which does not possess this inherent complexity, thus allowing tractable variational evaluation of the mean of the scattering amplitude T and of its absolute square $|T|^2$. Requiring no assumptions concerning the statistical nature of the scatterer, they proved that the average values of the individual integrals comprising a deterministic variational expression (e.g., for T) may themselves be combined to form a variationally invariant expression. Because evaluation of the mean of each integral is inherently more tractable than evaluation of the mean of the integral combination, this new variational approach is expected to allow more tractable calculation of the statistics of the scattered field. It has therefore been labeled the "stochastic" variational principle.[12]

Gray, Hart, and Farrell[13,14] performed the first application of this stochastic principle by treating a perfectly-conducting random rough surface model. This surface consists of parallel, infinitely-long hemicylinders randomly distributed over an infinite plane but constrained to be nonoverlapping. A plane electromagnetic wave is incident normally to and polarized parallel to the hemicylinder axes, thus reducing this problem to

a scalar, two-dimensional treatment. In the limit of a large number of Rayleigh hemicylinders they obtained the Born (i.e., the first order perturbation) approximation[15] for the mean absolute-squared scattering amplitude $\langle|T|^2\rangle$ in closed, analytic form. They then performed a variational improvement of the Born approximation, obtaining a closed-form variational solution, and noted a discrepancy between these results. This discrepancy was studied further by Krill and Farrell[16] who examined a special case of this surface in which only two Rayleigh hemicylinders are present. By comparing the exact solution to the corresponding Born and variational approximations of $\langle|T|^2\rangle$ for this surface, they demonstrated that the variational approximation is the more accurate one. This greater accuracy is achieved because the variational result accounts for multiple scattering, which is significant in this case, whereas the Born approximation does not.

Thus far, the stochastic variational expressions and their applications have been appropriate either for scalar (e.g., acoustic) fields or for vector fields with special scattering configurations.[13,16] Also, they have involved only homogeneous boundary conditions (e.g., perfect conductors). *Vector* stochastic variational principles will be derived in this paper to account for arbitrary electromagnetic field polarization and to allow treatment of conducting and dielectric random scatterers, which may be inhomogeneous and anisotropic. These new stochastic variational formulations express the statistical moments of the components of the vector scattering amplitude $\vec{T}$ as quotients of simpler statistical moments in a manner analogous to the scalar development of Hart and Farrell.[7]

2.2 Scope of This Treatment

As in the scalar case, the vector stochastic variational expressions are based on corresponding deterministic ones. Hence, in sections 3 and 4 we present deterministic variational principles for vector wave scattering from arbitrary conducting dielectrics, from a perfectly-conducting, rough planar surface, and from arbitrary perfect conductors. The formulations for the two configurations involving perfect conductors are distinguished by the choice of dyadic Green function. The vector variational formulations for scattering from arbitrary, nonplanar perfect conductor has appeared previously[10] and is included here for completeness. In section 5 we prove the variational invariance of the expressions for the arbitrary n^{th} statistical moments of the vector components of $\vec{T}$, and of their absolute squares, based on the deterministic formulations of the previous sections. Finally section 6 discusses the results and indicates some potential physical applications.

3. VARIATIONAL PRINCIPLE FOR CONDUCTING, DIELECTRIC SCATTERERS

In this section we shall derive in some detail a variational expression for the scattering of a plane electromagnetic wave by a finitely con-

ducting, dielectric volume, based on the vector wave equation for the electric field $\vec{E}$. It is an adaptation, to (vector) electromagnetic waves, of the scalar variational principle originally developed by Schwinger for the Schrödinger wave equation in quantum mechanics,[15] and, to our knowledge, has not appeared previously. (See, however, References 9 and 17-19 for related, but somewhat different, variational principles.) This expression will be recast into a stochastic variational principle in section 5.

Figure 1 illustrates the scattering problem. A plane wave with electric field $\vec{E}^i(\vec{r}) = A\hat{e}_i\, e^{i\vec{k}_i\cdot\vec{r}}$ is incident upon a scatterer (or scatterers) with free space permeability μ_0 and with generally inhomogeneous, anisotropic (i.e., dyadic) conductivity $\overline{\overline{\sigma}}(\vec{r})$ and permittivity $\overline{\overline{\varepsilon}}(\vec{r})$. The scattering volume V_0 is suspended in free space (ε_0, μ_0) and its geometrical characteristics, such as shape and orientation, are arbitrary. We wish to examine a vector polarization component of the scattering amplitude $\vec{T}$ along an arbitrary direction $\hat{e}_s$, i.e., $\hat{e}_s\cdot\vec{T}$, at an arbitrary position $\vec{r}$ in the far-field ($r \to \infty$, $r \equiv |\vec{r}|$).

The time-harmonic electric field $\vec{E}$ is governed by the vector wave equation,[10,20]

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}(\vec{r}) - k_0^2\vec{E}(\vec{r}) = \overline{\overline{U}}(\vec{r})\cdot\vec{E}(\vec{r}), \tag{1}$$

for which we have defined

$$\overline{\overline{U}}(\vec{r}) \equiv k_0^2\left[\frac{\overline{\overline{\varepsilon}}(\vec{r})}{\varepsilon_0} + \frac{i\overline{\overline{\sigma}}(\vec{r})}{\omega} - \overline{\overline{I}}\right], \tag{2}$$

where $k_0^2 = \omega^2\mu_0\varepsilon_0$, ω is the angular frequency, and $\overline{\overline{I}} \equiv \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}$ is the unit dyadic. The total field $\vec{E}$ can be expressed as the sum of the incident and scattered contributions, i.e., $\vec{E} = \vec{E}^i + \vec{E}^{sc}$, and $\vec{E}^{sc}$ satisfies the usual radiation condition at infinity,[20]

$$\lim_{r\to\infty} r\left[\vec{\nabla}\times\vec{E}^{sc}(\vec{r}) - ik_0\,\hat{r}\times\vec{E}^{sc}(\vec{r})\right] = 0 \tag{3}$$

for any localized current distribution on the scatterer, where $\hat{r} = \vec{r}/r$. This allows the asymptotic form ($r >> r'$) of $\vec{E}$ to be written as[9,10]

$$\vec{E}(\vec{r}) \to A\hat{e}_i\, e^{i\vec{k}_i\cdot\vec{r}} + A\vec{T}(\vec{k}_s,\vec{k}_i)e^{ik_0r}/r \tag{4}$$

for any localized scatterer. This equation defines the (angular-dependent) scattering amplitude $\vec{T}(\vec{k}_s,\vec{k}_i)$. We require $\vec{T}$ to be perpendicular to the scattered wave vector $\vec{k}_s$ since the scattered wave in the far field is transverse.[10,20]

In order to obtain an integral equation for $\vec{E}$ based on (1), we apply Green's Theorem for the vector wave equation[27] to yield

$$\vec{E}(\vec{r}) = \vec{E}^i(\vec{r}) + \int_V \overline{\overline{G}}_0(\vec{r},\vec{r}\,')\cdot\left[\overline{\overline{U}}(\vec{r}\,')\cdot\vec{E}(\vec{r}\,')\right]dV', \tag{5}$$

where the volume V is taken to be all of space, bounded only by the surface at infinity, on which the surface integrals involving $\vec{E}_{sc}$ vanish by virtue of the radiation condition. The work of Van Bladel[27] and Yaghjian[28] showed that (5) is valid even in the "source" region $\overline{\overline{U}}(\vec{r}\,') \neq 0$, provided that one chooses the dyadic Green function

$$\overline{\overline{G}}_0(\vec{r},\vec{r}\,') = \text{P.V. } \overline{\overline{G}}(\vec{r},\vec{r}\,') + \overline{\overline{L}}\delta(\vec{r}-\vec{r}\,'), \tag{6}$$

where $\overline{\overline{G}}(\vec{r},\vec{r}\,') = (\overline{\overline{I}}+\vec{\nabla}\vec{\nabla}/k_0^2)\exp(ik_0R)/4\pi R$ is the familiar free space dyadic Green function,[20] with $R \equiv |\vec{r}-\vec{r}\,'|$. They demonstrated this result by applying Green's Theorem while carefully excluding the volume around the singular point $\vec{r}\,' = \vec{r}$. The symbol P.V. in (6) indicates that the principle value is to be taken of an integral, such as that in (5), which involves $\overline{\overline{G}}$. The second term in (6) arises because of the nature of the singularity in the term $\vec{\nabla}\vec{\nabla}(1/R)$ at $R = 0$, and the dyadic $\overline{\overline{L}}$ depends on the shape of the excluded volume used in computing the principal value integral. Explicit expressions for $\overline{\overline{L}}$ for the more common excluded volume shapes have been tabulated in Reference 28. Note that $\overline{\overline{G}}_0$ reduces to $\overline{\overline{G}}$ when $\vec{r}$ is outside the source region; however, the more general form $\overline{\overline{G}}_0$ will be needed in the development of a variational principle.

Applying the asymptotic form of $\overline{\overline{G}}_0$ to (5) and observing (4) leads to an integral expression for $\vec{T}$,

$$\vec{T}(\vec{k}_s,\vec{k}_i) = \frac{1}{4\pi A}\vec{N}_1(\vec{k}_s,\vec{k}_i), \tag{7}$$

with

$$\vec{N}_1(\vec{k}_s,\vec{k}_i) \equiv \int_V \left(\bar{\bar{I}}_{\hat{r}}\, e^{-i\vec{k}_s\cdot\vec{r}'}\right)\cdot\left[\bar{\bar{U}}(\vec{r}')\cdot\vec{E}(\vec{r}')\right] dV', \qquad (8)$$

where $\vec{k}_s \equiv k_0\hat{r}$ and $\bar{\bar{I}}_{\hat{r}} \equiv \bar{\bar{I}} - \hat{r}\hat{r}$ is the projection operator onto the plane transverse to $\vec{k}_s$.

To complete our derivation of the variational expression for $\vec{T}$, we first premultiply (5) by a vector $[\tilde{\bar{\bar{U}}}(\vec{r})\cdot\tilde{\vec{E}}(\vec{r})]^{Tr}$, where the superscript Tr denotes matrix transposition. The operator $\tilde{\bar{\bar{U}}}$ and the field $\tilde{\vec{E}}$ are as yet undetermined except that, analogous to $\bar{\bar{U}}$, we choose $\tilde{\bar{\bar{U}}}$ to be independent of $\vec{E}$ and $\tilde{\vec{E}}$. The result is integrated over $\vec{r}\epsilon V$ to obtain

$$A = D(\vec{k}_s,\vec{k}_i)/N_2(\vec{k}_s,\vec{k}_i) \qquad (9)$$

with

$$N_2(\vec{k}_s,\vec{k}_i) = \int_V \left[\tilde{\bar{\bar{U}}}(\vec{r})\cdot\tilde{\vec{E}}(\vec{r})\right]^{Tr}\cdot\left(\hat{e}_i\, e^{i\vec{k}_i\cdot\vec{r}}\right) dV \qquad (10)$$

and

$$D(\vec{k}_s,\vec{k}_i) = \int_V \left[\tilde{\bar{\bar{U}}}(\vec{r})\cdot\tilde{\vec{E}}(\vec{r})\right]^{Tr}\cdot\vec{E}(\vec{r})dV$$

$$- \int_V dv \int_V dV' \left[\tilde{\bar{\bar{U}}}(\vec{r})\cdot\tilde{\vec{E}}(\vec{r})\right]^{Tr}\cdot\bar{\bar{G}}_0(\vec{r},\vec{r}')\cdot\left[\bar{\bar{U}}(\vec{r}')\cdot\vec{E}(\vec{r}')\right]. \qquad (11)$$

The evaluation of the double integral in (11) requires the dyadic Green function in the source region, and the definition given in (6) guarantees that this double integral is well defined. We combine (7) and (9) and premultiply the result by an arbitrary, constant vector $\hat{e}_s$ to yield

$$T(\vec{k}_s,\vec{k}_i) = \frac{1}{4\pi} N_1(\vec{k}_s,\vec{k}_i)N_2(\vec{k}_s,\vec{k}_i)/D(\vec{k}_s,\vec{k}_i), \qquad (12)$$

where $N_1 \equiv \hat{e}_s \cdot \vec{N}_1$, and where $T \equiv \hat{e}_s \cdot \vec{T}$ is the (experimentally measurable) component of the vector scattering amplitude along $\hat{e}_s$. We note that expressions (7), (9), and (12) are equalities only if the exact field $\vec{E}$, i.e., the solution to (5), is used in the evaluation of N_1, N_2, and D.

We will now examine what happens when approximate fields (which do not in general satisfy the wave equation and boundary conditions) are used. It will be shown that, for specific conditions on $\tilde{\vec{E}}$, expression (12) becomes variationally invariant in that the first-order variation of the functional T about the exact fields $\vec{E}$ and $\tilde{\vec{E}}$ vanishes,[9,21-23] i.e., $\delta T = 0$. This implies that first-order errors in N_1, N_2, and D which arise from replacing the exact fields $\vec{E}$ and $\tilde{\vec{E}}$ in (12) by approximate fields $\vec{E} + \delta\vec{E}$ and $\tilde{\vec{E}} + \delta\tilde{\vec{E}}$ do not lead to first order errors in the corresponding approximation for T. We observe that, upon using the same approximate field $\vec{E} + \delta\vec{E}$ to evaluate N_1, (7) also becomes an approximation to T, but with first-order errors.* To determine the conditions for invariance, the approximate fields are first inserted into (12) to obtain $T[\vec{E} + \delta\vec{E}, \tilde{\vec{E}} + \delta\tilde{\vec{E}}]$, where $T[\vec{E}, \tilde{\vec{E}}]$ denotes the exact expression. Expanding the difference $T[\vec{E} + \delta\vec{E}, \tilde{\vec{E}} + \delta\tilde{\vec{E}}] - T[\vec{E}, \tilde{\vec{E}}]$ in terms of the arbitrary variations $\delta\vec{E}$ and $\delta\tilde{\vec{E}}$, retaining only first-order terms, and dividing the result by the exact expression (12), one may show that the variation[21] δT is

$$\delta T/T = \delta N_1/N_1 + \delta N_2/N_2 - \delta D/D, \tag{13}$$

where δN_1, δN_2, and δD are the variations of N_1, N_2, and D, respectively.

The condition $\delta T = 0$ and the arbitrariness of the variation $\delta\tilde{\vec{E}}^{Tr}$ in (13) identifies the Euler equation[20,22] for $\vec{E}$,

$$\vec{E}(\vec{r}) = A\hat{e}_i\, e^{i\vec{k}_i \cdot \vec{r}} + \int_V dV'\, \overline{\overline{G}}_0(\vec{r},\vec{r}') \cdot \left[\overline{\overline{U}}(\vec{r}') \cdot \vec{E}(\vec{r}')\right], \quad \vec{r} \epsilon V_0 \tag{14}$$

and the arbitrariness of the variation $\delta\vec{E}$ identifies (after taking the transpose) the Euler equation for $\tilde{\vec{E}}$,

*Equations (7) and (12) lead to different approximations for T because the ratio (D/N_2) is not equal to the amplitude of the incident wave when approximate fields are used.

$$\left[\bar{\bar{U}}^{Tr}(\vec{r}\,')\right]^{-1}\cdot\tilde{\bar{\bar{U}}}(\vec{r}\,')\cdot\tilde{\vec{E}}(\vec{r}\,') = \frac{D}{N_1}\,\bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s\, e^{-i\vec{k}_s\cdot\vec{r}\,'} + \int_V dV\, \bar{\bar{G}}_0{}^{Tr}(\vec{r},\vec{r}\,')\cdot\tilde{\bar{\bar{U}}}(\vec{r})\cdot\tilde{\vec{E}}(\vec{r}),\ \vec{r}\,'\varepsilon V_0 \tag{15}$$

where the superscript -1 denotes matrix inversion. Observe that the Euler equations determine $\vec{E}$ and $\tilde{\vec{E}}$ only within the volume V_0 where $\bar{\bar{U}}$ is non-zero, i.e., the volume of the scatterer. (Of course, V_0 may coincide with V, e.g., for a scatterer with "soft" boundaries.) Thus, the Euler equation (14) for $\vec{E}$ is identical to the wave equation (5) only within V_0. This is consistent with the observation that the scattering amplitude (7) is completely determined by the field within the scatterer.

Conversely, provided solutions for $\vec{E}$ and $\tilde{\vec{E}}$ exist,[22,23] these Euler equations imply the vanishing of δT. Thus, under those conditions which the Euler equations may be satisfied, (12) is variationally invariant, in that first order variations about the exact solutions $\vec{E}$ and $\tilde{\vec{E}}$ are zero. (See References 22 and 23 for further discussion of these conditions.)

For our purposes it is convenient to choose the undetermined operator $\tilde{\bar{\bar{U}}}$ to equal the transpose of $\bar{\bar{U}}$ so that $\tilde{\bar{\bar{U}}} = \bar{\bar{U}}^{Tr}$. The Euler equation for $\vec{E}$ then reduces to

$$\tilde{\vec{E}}(\vec{r}\,') = \frac{D}{N_1}\,\bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s\, e^{-i\vec{k}_s\cdot\vec{r}\,'} + \int_V dV\, \bar{\bar{G}}_0(\vec{r}\,',\vec{r})\cdot\left[\bar{\bar{U}}^{Tr}(\vec{r})\cdot\tilde{\vec{E}}(\vec{r})\right] \tag{16}$$

for $\vec{r}\,'\varepsilon V_0$, where we have used the symmetry properties of $\bar{\bar{G}}_0$.[20] This equation is now in the form of an integral wave equation analogous to (5). An "adjoint" scattering configuration (Figure 2) may thus be identified from (16), which is related to the original problem by interchanging the parameters $\hat{e}_i \leftrightarrow \bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s$, $\vec{k}_i \leftrightarrow -\vec{k}_s$, and $\bar{\bar{U}} \leftrightarrow \bar{\bar{U}}^{Tr}$. If we indicate the parameters characterizing the original field $\vec{E}$ explicitly as $\vec{E}(A, \vec{k}_s, \vec{k}_i, \bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s, \hat{e}_i; \bar{\bar{U}})$, then these interchanges suggest an adjoint field solution $\tilde{\vec{E}}$ to the wave equation for the configuration in Figure 2, of the form $\tilde{\vec{E}} = \vec{E}(A, -\vec{k}_i, -\vec{k}_s, \hat{e}_i, \bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s; \bar{\bar{U}}^{Tr})$.[24] By deriving the adjoint quantities $\tilde{N}_1$, $\tilde{N}_2$, and $\tilde{D}$ for $\tilde{\vec{E}}$, analogous to (7) through (11), one observes that $A = \tilde{D}/\tilde{N}_2 = D/N_1$ since $\tilde{D} = D$ and $N_1 = \tilde{N}_2$. These relationships show that $T = \tilde{T}$ or more explicitly, $\hat{e}_s\cdot\vec{T}(\vec{k}_s, \vec{k}_i, \bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s, \hat{e}_i; \bar{\bar{U}}) = \hat{e}_i\cdot\vec{T}(-\vec{k}_i, -\vec{k}_s, \hat{e}_i, \bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s; \bar{\bar{U}}^{Tr})$, which is in the nature of a reciprocity relation.

4. VARIATIONAL EXPRESSIONS FOR PERFECT CONDUCTORS

Analysis of scattering from systems with high conductivities is often simplified by assuming the scatterers, e.g., rough metallic surfaces, to be perfect conductors.[2,25] In this section we shall present two variational principles for perfect conductors, distinguished by the type of dyadic Green function chosen for the integral field equations. The first formulation is specifically applicable to rough planar scattering surfaces. The second formulation, advantageous for nonplanar scatterers, has appeared elsewhere[10] and is included for completeness. As for the previous section, the principles derived here will be recast into forms suitable for stochastic systems in section 5.

4.1 Variational Principle for Perfectly Conducting, Planar Rough Surfaces

The scattering configuration is illustrated in Figure 3 in which a plane wave is incident upon an infinite, perfectly conducting, arbitrarily rough planar surface S_0 which we will restrict to lie on or above the z = 0 plane. In the linear, homogeneous, isotropic medium (μ, ε) above S_0 the wave equation for the electric field $\vec{E}$ is the homogeneous vector equation

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}(\vec{r}) - k^2\vec{E}(\vec{r}) = 0, \tag{17}$$

with $k^2 = \omega^2\mu\varepsilon$. On S_0 the field $\vec{E}$ is subject to the boundary condition $\hat{n}\times\vec{E}(\vec{r}) = 0$, $\vec{r}\varepsilon S_0$, where $\hat{n}$ is the outward normal from S_0. On the surface at infinity $\vec{E}$ is subject to the radiation condition (3).

To formulate the integral equation, we choose the half-space dyadic Green function of the first kind $\overline{\overline{G}}_1$, which satisfies the boundary condition $\hat{z}\times\overline{\overline{G}}_1(\vec{r},\vec{r}\,') = 0$ when z = 0 or z' = 0. This function has the form[20]

$$\overline{\overline{G}}_1(\vec{r},\vec{r}\,') = \left\{\overline{\overline{I}} - \frac{1}{k^2}\vec{\nabla}\vec{\nabla}'\right\}[G(\vec{r},\vec{r}\,') - G(\vec{r},\vec{r}_0\,')] + 2\hat{z}\hat{z}\,G(\vec{r},\vec{r}_0\,'), \tag{18a}$$

where $\vec{\nabla}$ and $\vec{\nabla}'$ are gradient operators over $\vec{r}$ and $\vec{r}\,'$, $G(\vec{r},\vec{r}\,') = \exp(ikR)/4\pi R$, and the image source point $\vec{r}_0\,'$ is related to $\vec{r}\,'$ by $\vec{r}_0\,' = \vec{r}\,' - 2z'\hat{z}$. Equivalently, $\overline{\overline{G}}_1(\vec{r},\vec{r}\,')$ may be expressed in terms of the free space dyadic Green function $\overline{\overline{G}}$ defined in section 3 as[4]

$$\overline{\overline{G}}_1(\vec{r},\vec{r}\,') = \overline{\overline{G}}(\vec{r},\vec{r}\,') - \overline{\overline{G}}(\vec{r},\vec{r}_0\,')\cdot\overline{\overline{R}}_{\hat{z}} \tag{18b}$$

in which $\overline{\overline{R}}_{\hat{z}} \equiv \overline{\overline{I}} - 2\hat{z}\hat{z}$ is the reflection operator about the z = 0 plane (note $\vec{r}_0' = \overline{\overline{R}}_{\hat{z}} \cdot \vec{r}'$). The first form (18a) exhibits clearly the boundary condition satisfied by $\overline{\overline{G}}_1$, and the second form (18b) illustrates the separate field contributions arising from a source and its image in the presence of a planar scattering surface.

By proper application of Green's Theorem to (17) in the volume bounded below by S_0 and above by the surface at infinity, one obtains the integral equation

$$\vec{E}(\vec{r}) = \vec{E}^i(\vec{r}) + \vec{E}^r(\vec{r}) - i\omega\mu \int_{S_0} \overline{\overline{G}}_1(\vec{r},\vec{r}')\cdot\vec{K}(\vec{r}')dS', \tag{19}$$

where $\vec{K}(\vec{r})$ is the surface current defined by

$$\vec{K}(\vec{r}) = \hat{n}\times\vec{H}(\vec{r}) = \frac{-i}{\omega\mu}\,\hat{n}\times\vec{\nabla}\times\vec{E}(\vec{r}),\ \vec{r}\varepsilon S_0,$$

and the boundary conditions have been applied. Here, $\vec{E}^i = A\hat{e}_i\, e^{i\vec{k}_i\cdot\vec{r}}$ is the prescribed incident plane wave, and $\vec{E}^r = -A\overline{\overline{R}}_{\hat{z}}\cdot\hat{e}_i\, e^{i(\overline{\overline{R}}_{\hat{z}}\cdot\vec{k}_i)\cdot\vec{r}}$ is the plane wave which would be specularly reflected from a perfectly conducting plane surface at z = 0.

Equation (19) is valid when $\vec{r}$ is not on the surface S_0. To evaluate the field on the surface [which will be necessary in order to obtain the integral equation (26) for $\vec{K}$], care must be exercised in view of the singular behavior of $\overline{\overline{G}}_1$ at R = 0. The field on the surface is defined as the limit of (19) as $\vec{r}$ approaches S_0, in analogy to Reference 9, Ch. 9. This limit can be verified to be finite. We observe that the choice of $\overline{\overline{G}}_1$ results in integration over only the rough region of S_0. By restricting the surface roughness to a finite, but possibly large, portion of the infinite surface S_0, the asymptotic form of (19) becomes

$$\vec{E}(\vec{r}) \xrightarrow[r>>r']{} \vec{E}^i + \vec{E}^r - A\,\vec{T}\,\frac{e^{ikr}}{r}, \tag{20}$$

where

$$\vec{T}(\vec{k}_s,\vec{k}_i) = \frac{i\omega\mu}{4\pi A}\,\vec{N}_1(\vec{k}_s,\vec{k}_i) \tag{21}$$

and

$$N_1(\vec{k}_s,\vec{k}_i) = \int_{S_0} dS\ \bar{\bar{I}}_{\hat{r}}\cdot\left[\bar{\bar{I}}\ e^{-i\vec{k}_s\cdot\vec{r}\,'} - \bar{\bar{R}}_{\hat{z}}\ e^{-i(\bar{\bar{R}}_{\hat{z}}\cdot\vec{k}_s)\cdot\vec{r}\,'}\right]\cdot\vec{K}(\vec{r}\,'). \tag{22}$$

The amplitude A of the incident wave may be eliminated from (21) by first introducing an "adjoint current" $\tilde{\vec{K}}(\vec{r})$ which we assume to be confined to S_0, such that $\tilde{\vec{K}}(\vec{r}) = 0$, $\vec{r}\notin S_0$ and $\hat{n}\cdot\tilde{\vec{K}}(\vec{r}) = 0$, $\vec{r}\in S_0$. Premultiplying (19) by $\tilde{\vec{K}}(\vec{r})$, integrating over S_0, and recognizing the boundary condition for $\vec{E}$ on S_0, we have

$$0 = \int_{S_0} dS\ \tilde{\vec{K}}(\vec{r})\cdot\vec{E}(\vec{r}) = AN_2(\vec{k}_s,\vec{k}_i) - i\omega\mu\ D(\vec{k}_s,\vec{k}_i), \tag{23}$$

where

$$N_2(\vec{k}_s,\vec{k}_i) \equiv \int_{S_0} dS\ \tilde{\vec{K}}(\vec{r})\cdot\left[\hat{e}_i\ e^{i\vec{k}_i\cdot\vec{r}} - \bar{\bar{R}}_{\hat{z}}\cdot\hat{e}_i\ e^{i(\bar{\bar{R}}_{\hat{z}}\cdot\vec{k}_i)\cdot\vec{r}}\right] \tag{24}$$

and

$$D(\vec{k}_s,\vec{k}_i) \equiv \int_{S_0} dS \int_{S_0} dS'\ \tilde{\vec{K}}(\vec{r})\cdot\bar{\bar{G}}_1(\vec{r},\vec{r}\,')\cdot\vec{K}(\vec{r}\,'). \tag{25}$$

In accordance with the discussion following the integral equation (19), D in (25) is to be evaluated by first performing the $\vec{r}\,'$ integration with $\vec{r}$ above S_0, and then letting $\vec{r}$ approach S_0 to perform the $\vec{r}$ integration. Combining (23) with (21) and defining the projections $T = \hat{e}_s\cdot\vec{T}$ and $N_1 = \hat{e}_s\cdot\vec{N}_1$ yields the familiar form (12), which is now a functional of the currents $\vec{K}$ and $\tilde{\vec{K}}$.

To identify the conditions for variational invariance, i.e., $\delta T = 0$, we perform the variation of T with respect to $\vec{K}$ and $\tilde{\vec{K}}$, in which the variations $\delta\vec{K}$ and $\delta\tilde{\vec{K}}$ will be restricted to S_0, with $\hat{n}\cdot\delta\vec{K} = 0$ and $\hat{n}\cdot\delta\tilde{\vec{K}} = 0$, but are otherwise arbitrary. The condition $\delta T = 0$ yields the Euler equations

$$A\ \bar{\bar{I}}_{\hat{n}}\cdot\left[\hat{e}_i\ e^{i\vec{k}_i\cdot\vec{r}} - \bar{\bar{R}}_{\hat{z}}\cdot\hat{e}_i\ e^{i(\bar{\bar{R}}_{\hat{z}}\cdot\vec{k}_i)\cdot\vec{r}}\right] = i\omega\mu\int_{S_0} dS'\ \bar{\bar{I}}_{\hat{n}}\cdot\bar{\bar{G}}_1(\vec{r},\vec{r}\,')\cdot\bar{\bar{I}}_{\hat{n}'}\cdot\vec{K}(\vec{r}\,'),\quad \vec{r}\in S_0 \tag{26}$$

and

$$\frac{D}{N_1}\,\bar{\bar{I}}_{\hat{n}}\cdot\left(\bar{\bar{I}}\,e^{-i\vec{k}_s\cdot\vec{r}\,'}-\bar{\bar{R}}_{\hat{z}}\,e^{-i(\bar{\bar{R}}_{\hat{z}}\cdot\vec{k}_s)\cdot\vec{r}\,'}\right)\cdot\bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s=\int_{S_0}dS\;\bar{\bar{I}}_{\hat{n}'}\cdot\bar{\bar{G}}_1(\vec{r}\,',\vec{r})\cdot\bar{\bar{I}}_{\hat{n}}\cdot\tilde{\vec{K}}(\vec{r}),\quad \vec{r}\,'\epsilon S_0 \tag{27}$$

where we have introduced the projection operator $\bar{\bar{I}}_{\hat{n}} \equiv \bar{\bar{I}} - \hat{n}\hat{n}$ onto S_0 to emphasize the tangential nature of the currents and their variations, [e.g., $\bar{\bar{I}}_{\hat{n}}\cdot\tilde{\vec{K}}(\vec{r}) = \tilde{\vec{K}}(\vec{r})$].

Equation (26) is just the equation for the current $\vec{K}$ on S_0, as we readily deduce from (19), while (27) is the corresponding equation for $\tilde{\vec{K}}$. Conversely, when solutions to the Euler equations (26) and (27) exist, then the functional T is stationary about the exact currents $\vec{K}$ and $\tilde{\vec{K}}$. Note that the left side of (27) may be identified as the tangential projection onto S_0 of the incident and specular plane waves for the adjoint problem shown in Figure 4. As in the case of scattering from conducting dielectrics in section 3, we may demonstrate that the function $\tilde{\vec{E}} = \vec{E}(A, -\vec{k}_i, -\vec{k}_s, \hat{e}_i, \bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s)$, where $\vec{E}(A, \vec{k}_s, \vec{k}_i, \bar{\bar{I}}_{\hat{r}}\cdot\hat{e}_s, \hat{e}_i)$ is the original field, satisfies the adjoint field equation and implies $(i\omega\mu)^{-1}\,\tilde{D}/\tilde{N}_2 = (i\omega\mu)^{-1}\,D/N_1 = A$.

4.2 Variational Principle for Closed, Perfectly Conducting Scatterers

The previous "half-space" variational principle is advantageous for a planar rough scattering surface because the integrations in (12) will be non-zero over only the rough portion of the surface S_0. However, when S_0 is closed, a scattering expression based on the free space dyadic Green function (6) is more convenient.[10] The variational principle for the configuration in Figure 5 has the standard form (12),[10] where now

$$N_1(\vec{k}_s,\vec{k}_i)=\oint_{S_0} e^{-i\vec{k}_s\cdot\vec{r}\,'}\,\hat{e}_s\cdot\bar{\bar{I}}_{\hat{r}}\cdot\vec{K}(\vec{r}\,')dS', \tag{28}$$

$$N_2(\vec{k}_s,\vec{k}_i)=\oint_{S_0}\tilde{\vec{K}}(\vec{r})\cdot\hat{e}_i\,e^{i\vec{k}_i\cdot\vec{r}}\,dS, \tag{29}$$

and

$$D(\vec{k}_s,\vec{k}_i) = \oint_{S_0} dS \oint_{S_0} dS' \; \tilde{\vec{K}}(\vec{r})\cdot\bar{\bar{G}}_0(\vec{r},\vec{r}\,')\cdot\vec{K}(\vec{r}\,'). \tag{30}$$

The integrals in (30) are defined by the same limiting procedure used in (25). Straight-forward variation shows the Euler equations for $\vec{K}$ and $\tilde{\vec{K}}$ to have the form (26) and (27), but without the explicit appearance of specular terms.

5. STOCHASTIC VARIATIONAL FORMULATIONS

The variational principles of the previous sections will now be recast into specific forms expressing statistical moments of an arbitrary component $T \equiv \hat{e}_s\cdot\vec{T}$ of the scattering amplitude $\vec{T}$. Recall that each of the variational expressions has the general form

$$T = \frac{1}{4\pi}\,\frac{N_1 N_2}{D}, \tag{31}$$

with the appropriate integrals N_1, N_2, and D. Following the procedure of Hart and Farrell[7] we shall use the nonstochastic nature of the amplitudes of the incident plane waves to derive exact expressions for the statistical moments of T and $|T|^2$ in terms of the corresponding moments of N_1, N_2, and D. More importantly, the expressions will be proven to be variationally invariant as a direct result of the invariance of the expression (31) for T.[7]

We begin by noting that for each scattering configuration equations of the form

$$A\,T = \frac{b}{4\pi}\,N_1 \tag{32}$$

and

$$A\,N_2 = bD \tag{33}$$

were derived, where $b = i\omega\mu$ for perfectly conducting scatterers and $b = 1$ for conducting dielectrics. In addition, our choice of solution $\tilde{\vec{E}}$ to the adjoint wave equation in the previous sections led to $AN_1 = bD$. If the scatterers fluctuate randomly, e.g., in shape, position, orientation, or material parameters, the expressions (31), (32), and (33) remain valid and exact, provided that we use the exact fields or currents on or within the scatterers. Since A is the nonstochastic amplitude of the incident plane wave (and since b is nonstochastic), the following relations may be obtained,

$$\langle T^n\rangle/\langle N_1{}^n\rangle = \langle T^n/N_1{}^n\rangle = T^n/N_1{}^n = b^n/(4\pi A)^n, \tag{34}$$

$$\langle N_2{}^n\rangle/\langle D^n\rangle = \langle N_2{}^n/D^n\rangle = N_2{}^n/D^n = b^n/A^n, \tag{35}$$

and

$$\langle N_1{}^n\rangle/\langle D^n\rangle = \langle N_1{}^n/D^n\rangle = N_1{}^n/D^n = b^n/A^n, \tag{36}$$

where $\langle\ \rangle$ denotes the average and the integer n is arbitrary. Combining (34) and (35) results in

$$\langle T^n\rangle = \left(\frac{1}{4\pi}\right)^n \langle N_1{}^n\rangle\langle N_2{}^n\rangle/\langle D^n\rangle. \tag{37}$$

This is an exact expression for $\langle T^n\rangle$ when the exact fields $\vec{E}$ and $\overset{\Rightarrow}{E}$, or exact currents $\vec{K}$ and $\overset{\Rightarrow}{K}$, are inserted in the appropriate integrals.

We will show that if $\delta T = 0$, then $\delta\langle T^n\rangle = 0$, that is, (37) is stationary about the exact fields or currents. First define

$$\langle T'^n\rangle \equiv \left(\frac{1}{4\pi}\right)^n \frac{\langle(N_1+\delta N_1)^n\rangle\langle(N_2+\delta N_2)^n\rangle}{\langle(D+\delta D)^n\rangle} \tag{38}$$

where, for example, $N_1 + \delta N_1$ is the result of approximating the exact current $\vec{K}$ by the trial function $\vec{K}' = \vec{K} + \delta\vec{K}$. Thus, the trial approximations for $\vec{K}$ and $\overset{\Rightarrow}{K}$ (or $\vec{E}$ and $\overset{\Rightarrow}{E}$) introduce the errors δN_1, δN_2, and δD, which are due to the variations of these currents (or fields) from their exact values. Then, to the first order in δN_1, δN_2, and δD,

$$\begin{aligned}\delta\langle T^n\rangle/\langle T^n\rangle &\equiv \langle T'^n - T^n\rangle/\langle T^n\rangle \\ &= n\left\{\frac{\langle(\delta N_1/N_1)N_1{}^n\rangle}{\langle N_1{}^n\rangle} + \frac{\langle(\delta N_2/N_2)N_2{}^n\rangle}{\langle N_2{}^n\rangle} - \frac{\langle(\delta D/D)D^n\rangle}{\langle D^n\rangle}\right\}.\end{aligned} \tag{39}$$

Applying (35) and (36) through (39) results in

$$\delta\langle T^n\rangle/\langle T^n\rangle = n\langle D^n(\delta N_1/N_1 + \delta N_2/N_2 - \delta D/D)\rangle/\langle D^n\rangle \tag{40}$$

For variational principles with the form (31), the quantity in parentheses in (40) vanishes, and we have proven that

$$\delta\langle T^n\rangle = 0 \tag{41}$$

when Eq. (36) is used to express $\langle T^n \rangle$.

To calculate statistical moments of the differential scattering cross section, we may derive a general variational expression for $\langle |T|^{2n} \rangle$. Applying (32) and (33) the appropriate expression is

$$\langle |T|^{2n} \rangle = \left(\frac{1}{4\pi}\right)^{2n} \langle |N_1|^{2n} \rangle \langle |N_2|^{2n} \rangle / \langle |D|^{2n} \rangle. \tag{42}$$

Proceeding as before, we readily show that

$$\delta \langle |T|^{2n} \rangle = 0, \tag{43}$$

that is, first order variations of (42) about the exact fields or currents vanish.

Under the usual conditions for convergence, note that the Fourier transform of the probability density function $f(T)$ of the component $T = \hat{e}_s \cdot \vec{T}$ may be expressed as the sum of statistical moments,[26]

$$\mathcal{F}\{f(T) = \sum_{n=0}^{\infty} \frac{(i\nu)^n}{n!} \langle T^n \rangle, \tag{44}$$

where $\mathcal{F}$ denotes the Fourier transform over parameter ν. A similar expression may be written for $f(|T|^2)$ where $|T|^2$ is simply the differential scattering cross section $d\sigma/d\Omega$.[25] Thus, variational approximations for $f(T)$ and $f(|T|^2)$ may be obtained from the variational calculations of successive statistical moments of T and $|T|^2$.

Variational principles of the form (36) or (42) are readily observed to be inherently more tractable than corresponding variational expressions obtained directly from (31), i.e.,

$$\langle T^n \rangle = \left(\frac{1}{4\pi}\right)^n \langle N_1{}^n N_2{}^n / D^n \rangle \tag{45}$$

and

$$\langle |T|^{2n} \rangle = \left(\frac{1}{4\pi}\right)^{2n} \langle |N_1|^{2n} |N_2|^{2n} / |D|^{2n} \rangle. \tag{46}$$

We see, for example, that (46) involves calculating the average of the complicated quotient $N_1{}^n N_2{}^n / D^n$, whereas (36) involves the quotient of the averages $\langle N_1{}^n \rangle$, $\langle N_2{}^n \rangle$, and $\langle D^n \rangle$, which are generally simpler to evaluate.

6. SUMMARY

The configurations for which stochastic variational expressions of the forms (36) and (42) have been derived (Figures 1, 3, and 5) are representative of a wide range of scattering problems. For example, precipitation and certain aerosols may be modelled as ensembles of conducting, dielectric scatterers, to which (8), (10), and (11) are immediately applicable, and the surface of the ocean may be modelled as a rough planar conductor for which (22), (24), and (25) are appropriate. Stochastic variational principles show promise in allowing application of the variational method to these and other random scattering problems. Future work will involve demonstration of their tractability by explicit calculation of the statistics of electromagnetic wave scattering from models of random volume and rough surface phenomena.

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APPENDIX

FIGURES

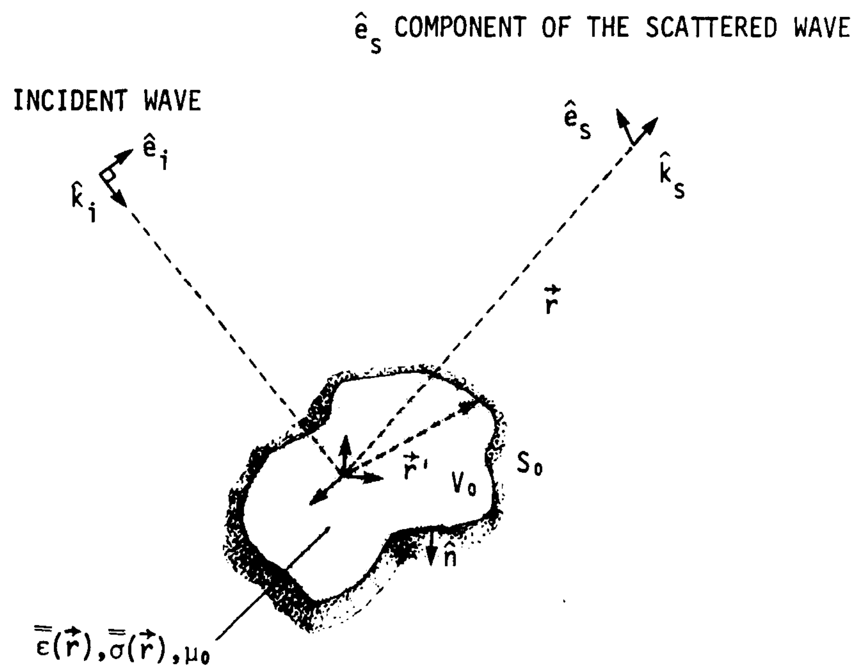

Figure 1. Geometry for Scattering from an Arbitrary Volume V_0 with Permittivity $\overline{\overline{\varepsilon}}(\vec{r})$ and Conductivity $\overline{\overline{\sigma}}(\vec{r})$ Suspended in Free Space

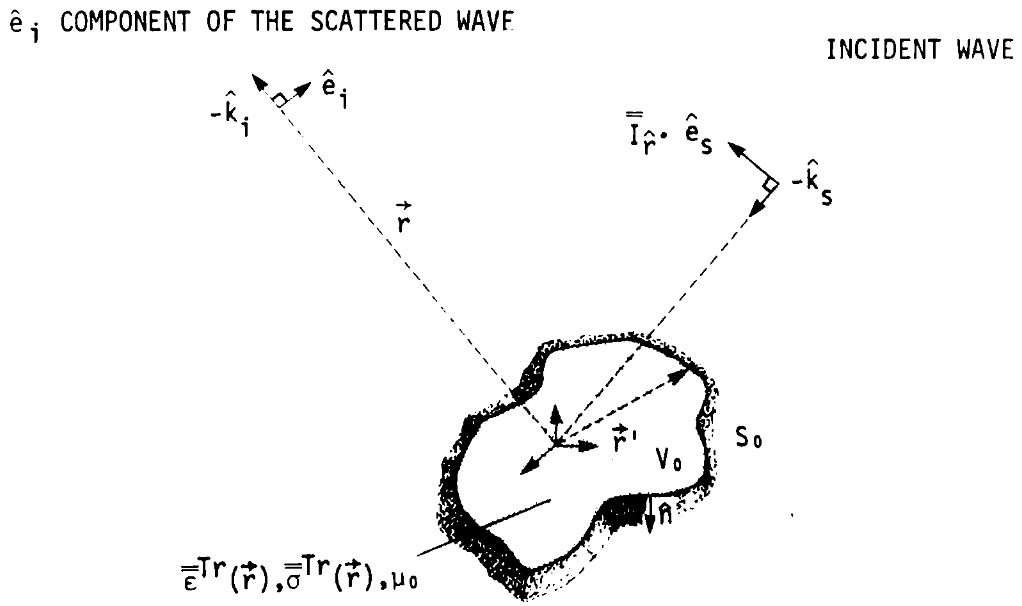

Figure 2. Counterpart to Figure 1 where Roles of Polarization and Propagation are Interchanged between Incident and Scattered Fields, and $\overline{\overline{\epsilon}}$ and $\overline{\overline{\sigma}}$ of the Scattered Volume are Transposed.

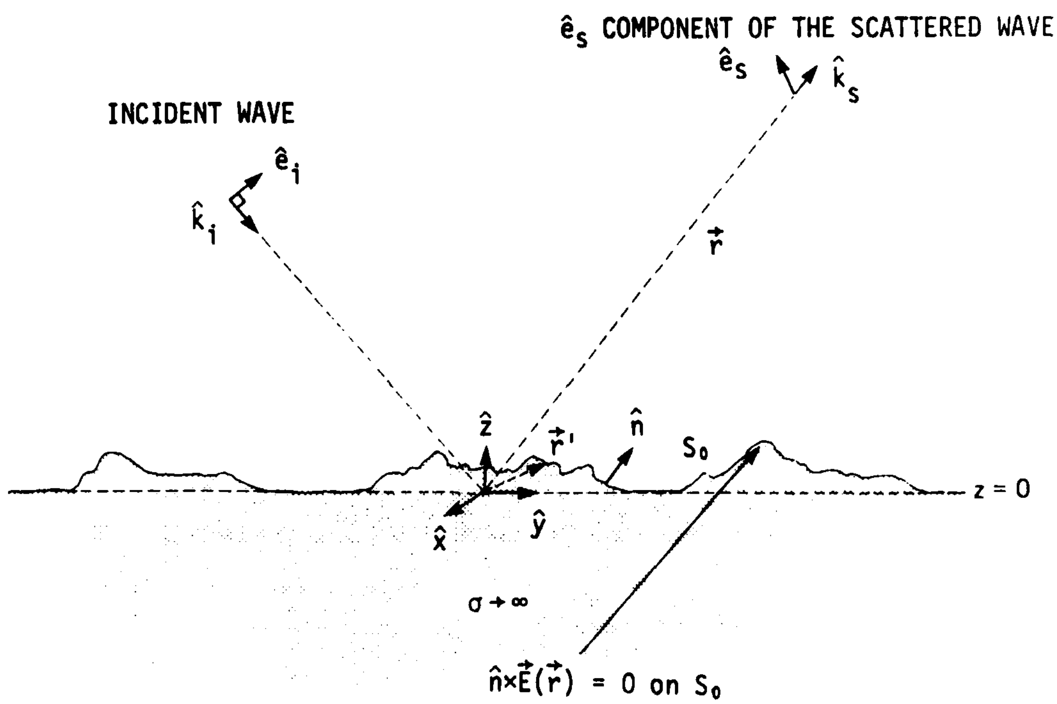

Figure 3. Scattering of a Plane Wave by a Perfectly Conducting Planar Surface with a Confined Roughness Region.

(The variational principle is derived for the nonspecular portion of the scattered field.)

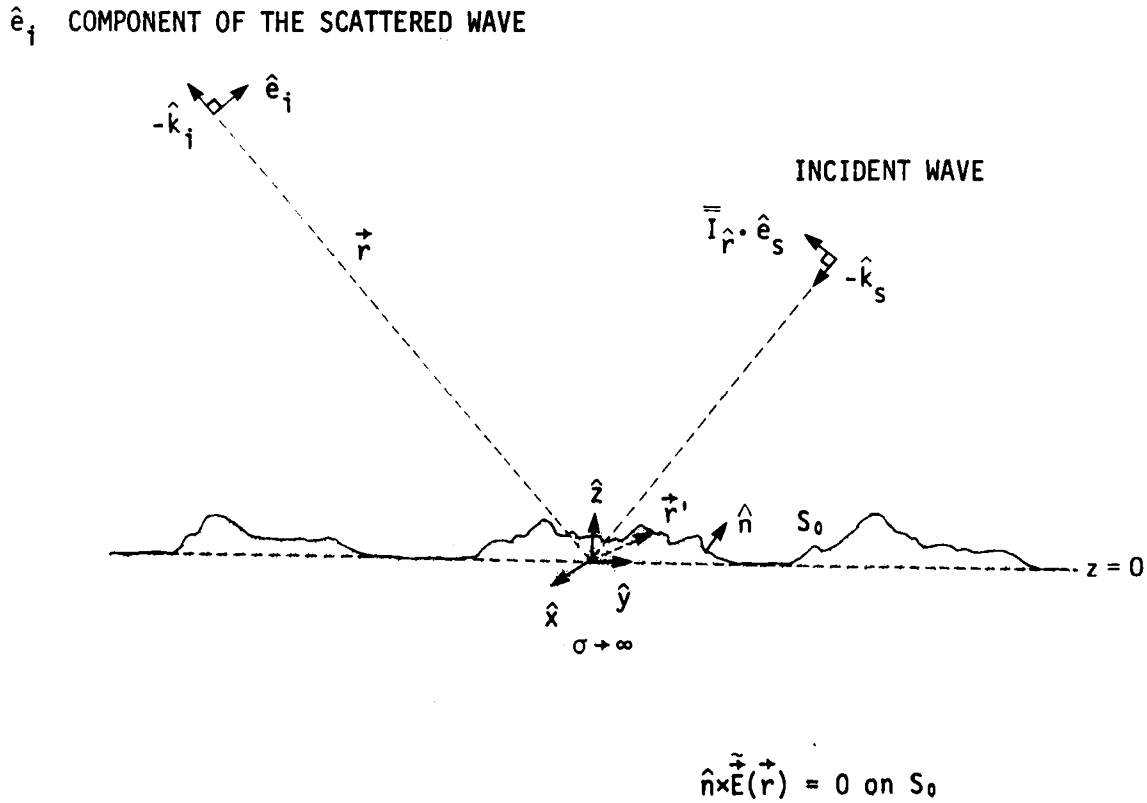

Figure 4. Adjoint Counterpart of Figure 3 where the Roles of the Incident and Scattered Wave Parameters are Interchanged.

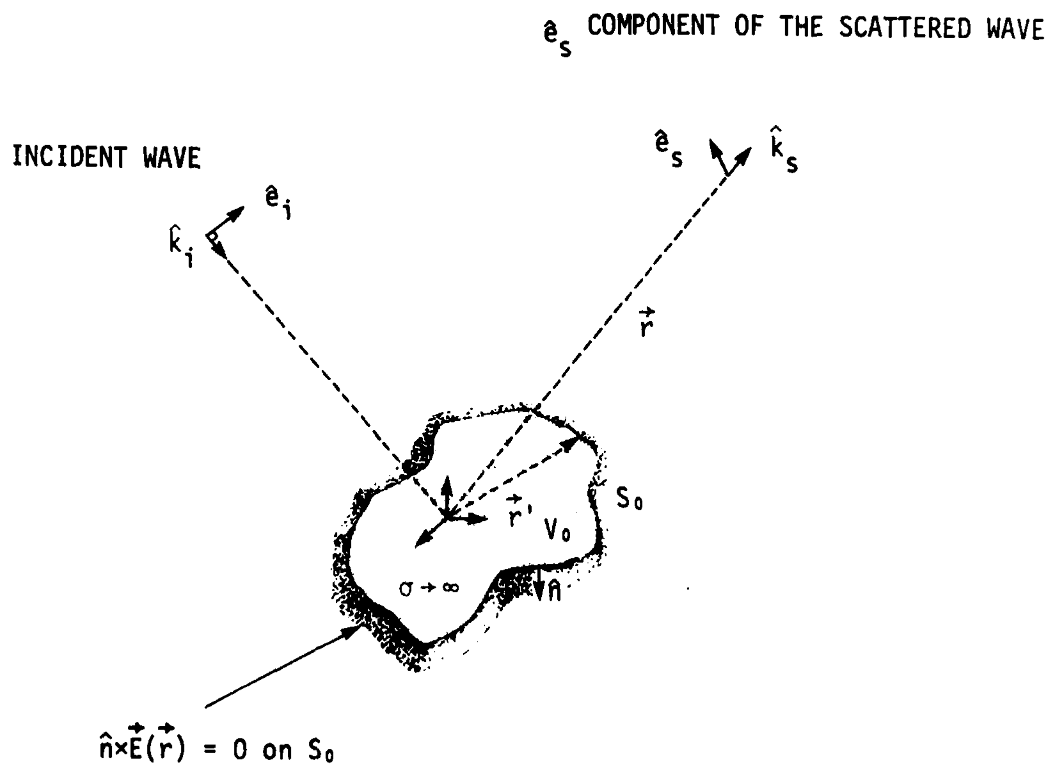

Figure 5. A Plane Wave Impinging on a Closed, Perfectly Conducting Sphere.

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